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Pair creation in cosmology when electromagnetic fields are present

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Abstract. The amount of particle pairs created in a 3-flat Robertson–Walker Universe with an expansion law for the early Universe is calculated exactly when a homogeneous electromagnetic field is present. Under some restrictions a time-dependent particle creation rate is found. Finally we show that the low-frequency part of the cosmological 2·7 K background radiation can be identified with the stationary electromagnetic field discussed before. According to this a large amount of particles of the order of the number of particles in the Universe should be created out of the vacuum in the immediate neighbourhood of the 'big bang'.

1. Introduction

In the existing papers about particle creation in the expanding Universe the generation of particles out of the vacuum is usually discussed by embedding the *free* particle fields in the time-dependent classical cosmological metrics. It follows the result that the number of created particles is in general negligibly small in comparison with the number of particles in the Universe, according to which particle creation out of the vacuum would have no cosmological importance. Recently, however, Müller *et al* (1978) have shown that, in the case of the Robertson–Walker line elements the number of created particles grows very rapidly for expansion laws resulting, via the Friedman equations, from equations of state of the extreme form $p \stackrel{<}{\to} \rho$.

For the existence of such equations of state for the cosmological substratum additional interaction fields besides the gravitational one are necessary, e.g. electromagnetic fields or even strong interaction fields. And, indeed, in view of the observational situation of the 2·7 K background radiation strong electromagnetic fields must have existed in the earlier stages of the Universe. However, these fields do not influence only the expansion law via the Friedman equations—as considered by Müller *et al*—but enter the dynamical equations of the particle field in question directly as external fields too. In consequence of this fact an important amplification of particle creation is to be expected.

Therefore the purpose of this paper is the examination of the pair creation process and the calculation of the number density of the created pairs when *both* an external electromagnetic field and a cosmological gravitational field (without a cosmological event horizon) are present. To be able to perform exact calculations we restrict ourselves to a homogeneous electric and magnetic field and to a 3-flat Robertson–

Walker metric describing the early Universe. Later the applicability of our results to the situation of strong cosmological electromagnetic radiation fields is discussed.

Our considerations are also of interest for the following reason. Whereas in the case of the homogeneous electromagnetic field alone a particle creation rate is definable which is constant (see Schwinger 1951), in the pure case of the Robertson–Walker Universe used by us no particle rate appears (see Audretsch and Schäfer 1978). Only in cosmological situations with cosmological event horizons can a pair creation rate be expected (see Gibbons and Hawking 1977). Furthermore, in the case of a pure homogeneous magnetic field no particle creation occurs. So it is an interesting point to combine the electromagnetic pair creation mechanism and the cosmological one and to look at what will happen.

2. External fields

For the gravitational field we take the cosmological 3-flat Robertson–Walker line element in conformally flat form:

$$ds^2 = \Omega^2(\eta)\{d\eta^2 - dx^2 - dy^2 - dz^2\} \qquad (-\infty < \eta, x, y, z < +\infty). \quad (2.1)$$

The corresponding Friedman equation takes the form (the prime denotes the derivative with respect to the conformal time η)

$$\Omega'^2 = \tfrac{8}{3}\pi G\rho\Omega^4 \quad (2.2)$$

(G is Newton's gravitational constant). For the energy density ρ of the cosmological substratum we set, considering the early, radiation-dominated phase of the Universe,

$$\rho = \sigma T^4 - 2\pi\epsilon^2\lambda_\pi^2 n^2 \quad (2.3)$$

where the first term represents the radiation field (T is its temperature, σ is the Stefan–Boltzmann constant) and the second one takes roughly into account the binding strong interaction forces between the nucleons (n is their number density, ϵ is the coupling constant of the strong interaction, λ_π is the Compton wavelength of the pion; for details see Dehnen and Hönl (1975)). With the usual adiabatic conservation laws (conservation of the number of nucleons and photons)

$$n\Omega^3 = A = \text{constant} \qquad T\Omega = B = \text{constant} \quad (2.4)$$

we are able to integrate equation (2.2) after inserting (2.3). We find

$$\Omega^2 = a^2 + b^2\eta^2 \quad (2.5)$$

with

$$a^2 = 2\pi\epsilon^2\lambda_\pi^2 A^2/\sigma B^4 \qquad b^2 = \tfrac{8}{3}\pi G\sigma B^4 \quad (2.5a)$$

where we have chosen the minimum value of Ω^2 at the time $\eta = 0$. We remark that the expansion law (2.5), used also by Audretsch and Schäfer (1978), is necessarily connected with an energy density behaviour according to (2.3) and (2.4).

The transition from (2.1) to the usual form of the 3-flat Robertson–Walker line element

$$ds^2 = dt^2 - \Omega^2(t)\{dx^2 + dy^2 + dz^2\} \quad (2.6)$$

yields the following connection between the conformal time η and the measurable cosmological time t with the property $\eta = 0 \leftrightarrow t = 0$:

$$t = \frac{a^2}{2b}\left\{\frac{b}{a}\eta\left[1+\left(\frac{b}{a}\eta\right)^2\right]^{1/2} + \ln\left(\frac{b}{a}\eta + \left[1+\left(\frac{b}{a}\eta\right)^2\right]^{1/2}\right)\right\}. \tag{2.7}$$

For small times $|\eta| \ll a/b$, i.e. $|t| \ll a^2/b$, we get

$$t = a\eta \qquad \Omega^2 = a^2\left[1+\left(\frac{b}{a^2}t\right)^2\right] \tag{2.7a}$$

and for large times $|\eta| \gg a/b$, corresponding to $|t| \gg a^2/2b$, we obtain

$$|t| = \tfrac{1}{2}b\eta^2 \qquad \Omega^2 = 2b|t|. \tag{2.7b}$$

Evidently for large absolute values of the time t we have the well-known contracting and expanding radiation universes (2.7b), which are connected for small absolute values of the time t by the parabolic expansion law (2.7a); the minimum value of Ω ($\Omega_{\min} = a$) is caused by the strong interaction forces according to (2.5a).

As external electromagnetic field we choose a homogeneous parallel electric and magnetic field. For this in the flat Minkowski space–time the 4-potential A_μ satisfying Maxwell's vacuum equations reads:

$$A_\mu = E\eta\delta^3_\mu - Hx\delta^2_\mu \qquad (E = \text{constant},\ H = \text{constant}) \tag{2.8}$$

whereby the electric and magnetic fields lie in the direction of the z axis. Because Maxwell's vacuum equations are conformally invariant and the metric (2.1) is conformally flat, the 4-potential (2.8) with covariant index position is also a solution for a homogeneous parallel electric and magnetic field in the case of the cosmological metric (2.1) (cf Stephani 1972). For the absolute values of the observable electric and magnetic field strengths

$$\mathscr{E} = |E|/\Omega^2 \qquad \mathscr{H} = |H|/\Omega^2. \tag{2.9}$$

3. Particle field

As dynamical equation for the particle field we take the conformally coupled Klein–Gordon equation:

$$[g^{\mu\nu}(\mathrm{i}\nabla_\mu - eA_\mu)(\mathrm{i}\nabla_\nu eA_\nu) - \tfrac{1}{6}R - m^2]\Phi = 0. \tag{3.1}$$

The scalar product is given by

$$(\Phi_1, \Phi_2) = \mathrm{i}\int_\sigma \Phi_1^* \overleftrightarrow{f}^\mu \Phi_2 \,\mathrm{d}\sigma_\mu \tag{3.2}$$

with

$$\overleftrightarrow{f}^\mu = \sqrt{-g}\, g^{\mu\nu}(\vec{\partial}_\nu + \mathrm{i}eA_\nu) - (\overleftarrow{\partial}_\nu - \mathrm{i}eA_\nu)\sqrt{-g}\, g^{\mu\nu}. \tag{3.2a}$$

4. Number density of the created particle pairs

For calculation of the particle pairs created out of the vacuum as a consequence of the external fields we have to solve the Klein–Gordon equation (3.1) using the 4-potential

(2.8) and the metric (2.1) with Ω according to (2.5). For this we make the separation ansatz

$$\Phi = \phi/\Omega \qquad \phi = f(\eta)g(x)\exp[-\mathrm{i}(yk_2+zk_3)] \tag{4.1}$$

with the constant wavenumbers k_2 and k_3. Hence we get from (3.1) immediately

$$\left[\frac{\partial^2}{\partial\eta^2}-\frac{\partial^2}{\partial x^2}+(k_2+eHx)^2+(k_3-eE\eta)^2+m^2(a^2+b^2\eta^2)\right]fg=0. \tag{4.2}$$

Using the abbreviations

$$\begin{aligned} &\gamma^2 = e^2E^2+m^2b^2 \\ &\xi = \eta - eEk_3/\gamma^2 \qquad \zeta = x + k_2/eH \end{aligned} \tag{4.3}$$

equation (4.2) can be written as

$$\left[\frac{\partial^2}{\partial\xi^2}-\frac{\partial^2}{\partial\zeta^2}+(eH)^2\zeta^2+\gamma^2\xi^2+\left(\frac{mb}{\gamma}\right)^2k_3^2+m^2a^2\right]fg=0. \tag{4.4}$$

Evidently with respect to the coordinate ζ equation (4.4) has the behaviour of the harmonic oscillator equation. Normalising the wavefunction ϕ according to (3.2) and (3.2*a*) we find the following two complete systems of solutions of equation (4.4), whereby the upper sign designates the first system and the lower sign the second system (cf Damour 1977):

$$ {}^{-}_{+}\phi_{\nu,k_2,k_3} = u_{\nu,k_2,k_3}D_{\mathrm{i}\lambda-\frac{1}{2}}(\pm[1-\mathrm{i}]\sqrt{\gamma}\,\xi) \tag{4.5a}$$

$$ {}^{+}_{-}\phi_{\nu,k_2,k_3} = u_{\nu,k_2,k_3}D_{-\mathrm{i}\lambda-\frac{1}{2}}(\pm[1+\mathrm{i}]\sqrt{\gamma}\,\xi) \tag{4.5b}$$

with the abbreviations

$$\begin{aligned} u_{\nu,k_2,k_3} &= (2\pi)^{-1}(2^\nu\nu!)^{-1/2}\left(\frac{e|H|}{2\pi\gamma}\right)^{1/4}\exp\left(-\frac{\pi}{4}\lambda\right)H_\nu(e|H|\zeta) \\ &\quad\times\exp(-\tfrac{1}{2}e|H|\zeta^2-\mathrm{i}(yk_2+zk_3)) \\ \lambda &= \frac{1}{2\gamma}\left[l^2+\left(\frac{mb}{\gamma}\right)^2k_3^2+m^2a^2\right] \\ l^2 &= 2(\nu+\tfrac{1}{2})e|H| \qquad \nu = 0, 1, 2, \ldots . \end{aligned} \tag{4.5c}$$

Here H_ν are the Hermite polynomials and $D_{\pm\mathrm{i}\lambda-\frac{1}{2}}$ are the parabolic cylinder functions (see e.g. Magnus *et al* 1966).

The solutions of the Klein–Gordon equation (3.1) given by (4.1) and (4.5) with the lower signs

$${}_{+-}\Phi = {}_{+-}\phi/\Omega \tag{4.6a}$$

represent *asymptotically* for $t\to-\infty$ (η, $\xi\to-\infty$), and the solutions with the upper signs

$${}^{-+}\Phi = {}^{-+}\phi/\Omega \tag{4.6b}$$

for $t\to+\infty$ (η, $\xi\to+\infty$), stationary energy eigensolutions; the energy eigenvalues with respect to the cosmological time t are $\pm m$, corresponding to particles (positive signs) and antiparticles (negative signs). This asymptotical particle definition is identical with that in the papers of Schäfer (1978) or Nikishov (1970) and will be used here too.

The number of created particle pairs per mode (ν, k_2, k_3) is given by (cf Damour 1977)

$$\hat{n}_{\nu,k_2,k_3}\Lambda = \sum_{\nu'} \int_{k_2'} \int_{k_3'} |({}^+\Phi_{\nu,k_2,k_3}, {}_-\Phi_{\nu',k_2',k_3'})|^2 \, \mathrm{d}k_2' \, \mathrm{d}k_3' \tag{4.7}$$

with

$$\Lambda = (2\pi)^2 \delta(k_2 - k_2)\delta(k_3 - k_3)$$

where Λ is identical with the coordinate area of the (y, z) plane. Inserting the functions (4.6a) and (4.6b) with the use of (4.5) into the relation (4.7), we obtain

$$\hat{n}_{\nu,k_2,k_3} = (2\pi)^{-2} \, \mathrm{e}^{-2\pi\lambda}. \tag{4.8}$$

Then the total number of pairs created per space coordinate volume has the form

$$\hat{n} = \sum_{\nu} \int_{k_2} \int_{k_3} \hat{n}_{\nu,k_2,k_3} \, \mathrm{d}k_2 \, \mathrm{d}k_3 \Big/ \int \mathrm{d}x. \tag{4.9}$$

Using (4.8), (4.5c) and (4.3) we get, with the relation $\mathrm{d}k_2 = -eH \, \mathrm{d}x$ (see § 5), from (4.9) finally:

$$\hat{n} = \frac{(e^2E^2 + m^2b^2)^{5/4}}{(2\pi)^3 mb} K \exp\left(-\frac{\pi m^2 a^2}{(e^2E^2 + m^2b^2)^{1/2}}\right)$$
$$K = \frac{\pi eH}{(e^2E^2 + m^2b^2)^{1/2}} \left(\sinh \frac{\pi eH}{(e^2E^2 + m^2b^2)^{1/2}}\right)^{-1}. \tag{4.10}$$

The observable number density of the created pairs in the asymptotic region ($t \to +\infty$) is given by

$$n = \hat{n}/\Omega^3. \tag{4.10a}$$

Evidently the factor K in formula (4.10) contains the influence of the magnetic field on the particle creation. One finds that

$$0 < K \leqslant 1. \tag{4.11}$$

That means that the magnetic field reduces the particle generation. Only in the case of $H \to 0$ has one $K \to 1$. On the other hand, we have the result that the electric field amplifies monotonically the particle creation and will be very effective for

$$e|E|/mb \gg 1. \tag{4.12}$$

Thus our supposition is confirmed, that non-gravitational external fields are able to increase the particle generation in comparison with the pure gravitational creation mechanism, especially in the case (4.12). The cosmological consequences of this fact will be discussed in § 6.

Finally, we note that the strong result for the number density of created particle pairs (4.10) does not consist of two additive parts representing the influence of the electromagnetic and of the gravitational field separately; evidently the creation process is a *coherent* effect of both fields. In the limiting case $E \to 0$, $H \to 0$ the pure gravitational result of Audretsch and Schäfer (1978) is obtained.

5. Particle creation rate

Until now no particle creation rate has been found. But because in the pure electromagnetic case a creation rate exists, we will consider under what conditions the construction of a particle creation rate from equation (4.10) is possible.

The functions (4.5) already have a quasi-classical behaviour with respect to time, which means the parabolic cylinder functions $D_{\pm i\lambda-\frac{1}{2}}$ go over into exponential functions with the classical action function as exponent (cf Schäfer 1978), for

$$|\xi| \gg \gamma^{-1/2} \qquad |\xi| \gg m^2a^2/\gamma^{3/2} \tag{5.1}$$

(apart from a (l^2, k_3^2)-dependent additional term on the right-hand side of the second relation, which is irrelevant for our considerations). According to this the particle creation happens around the 'time' $\xi = 0$ within the interval $1/\gamma^{1/2}$ or $m^2a^2/\gamma^{3/2}$, respectively. Then it follows from (4.3) that on the average the canonical momentum k_3 is created at the conformal time

$$\eta = eEk_3/\gamma^2. \tag{5.2a}$$

The analogous procedure for the ζ-dependent part of equation (4.5) yields the following connection between the x coordinate and the momentum k_2:

$$x = k_2/eH \tag{5.2b}$$

which has been used already for the transition from equation (4.9) to (4.10). Herewith we can reformulate equation (4.9) by substituting k_3 and k_2 and integrating over x. In this way we get

$$\hat{n} = \sum_{\nu} \int_{-\infty}^{+\infty} \frac{\gamma^2|H|}{|E|} \hat{n}_{\nu,\eta}\, d\eta. \tag{5.3}$$

Consequently we find the following *formal* creation rate using (4.8) and (4.3):

$$\frac{d}{d\eta}\hat{n} = \sum_{\nu} \frac{\gamma^2|H|}{|E|}\hat{n}_{\nu,\eta} = \frac{(e^2E^2+m^2b^2)^{3/2}}{(2\pi)^3 e|E|} K$$
$$\times \exp\left[-\pi m^2a^2(e^2E^2+m^2b^2)^{-1/2} - \pi\eta^2\left(\frac{mb}{eE}\right)^2 (e^2E^2+m^2b^2)^{1/2}\right] \tag{5.4}$$

with K according to (4.10). In the case $e|E|/mb \ll 1$ the value of (5.4) vanishes very rapidly.

The maximum value of the canonical momentum k_3 produced is still, according to (4.8) with respect to (4.5c), given sufficiently by:

$$|k_3|_{\max} \simeq \gamma^{3/2}/mb. \tag{5.5}$$

The interval for the particle creation is therefore, in view of (5.2a), determined by

$$|\eta|_{\max} \simeq \frac{e|E|}{mb}\gamma^{-1/2}. \tag{5.6}$$

This statement has physical meaning only if the value of (5.6) is larger than the creation time for the single particle pair given by the right-hand sides of (5.1). And only in this case does the particle creation rate (5.4) make sense, i.e. for

$$e|E|/2mb \gg 1 \qquad (eE/mb)^2 \gg ma^2/b. \tag{5.7}$$

Here, and with the use of (2.5), equation (5.4) results in the physically meaningful pair creation rate:

$$\frac{\mathrm{d}}{\mathrm{d}\eta}\hat{n}=\frac{(eE)^2}{(2\pi)^3}\frac{\pi H/E}{\sinh(\pi H/E)}\exp\left(-\frac{\pi m^2\Omega^2}{e|E|}\right). \tag{5.8}$$

From this relation we find for the particle pair creation rate per measurable space volume and per cosmological time unit:

$$\begin{aligned} r=\frac{1}{\Omega^3}\frac{\mathrm{d}}{\mathrm{d}t}\hat{n} &= \frac{(eE/\Omega^2)^2}{(2\pi)^3}\frac{\pi H/E}{\sinh(\pi H/E)}\exp\left(-\frac{\pi m^2\Omega^2}{e|E|}\right)\\ &= \frac{(e\mathscr{E})^2}{(2\pi)^3}\frac{\pi\mathscr{H}/\mathscr{E}}{\sinh(\pi\mathscr{H}/\mathscr{E})}\exp\left(-\frac{\pi m^2}{e\mathscr{E}}\right) \end{aligned} \tag{5.9}$$

where $\mathscr{E}$ and $\mathscr{H}$ are the absolute values of the measurable electric and magnetic field strengths according to (2.9). The electric field strength necessary for non-negligible particle creation is therefore:

$$\mathscr{E}\gtrsim m^2/e\mathrel{\hat{=}}10^{16}\ \mathrm{V\ cm^{-1}} \tag{5.9a}$$

(m is the mass of the proton), which corresponds to the time limits $\eta_{\max}$ or $t_{\max}$ (see (5.6) or (6.11)).

We note that the lower line of (5.9) is identical with the expression for the particle pair creation in a homogeneous magnetic and electric field within the flat Minkowski space–time (cf Damour 1977), when the cosmological time dependence of the field strength is taken into account. Clearly this result is to be expected in the case of *strong* electromagnetic field strength (see (5.7)).

In the case of *weak* electromagnetic fields, i.e. $e|E|/mb\ll 1$ and $e|H|/mb\ll 1$ (cf the exact solution (4.10)), $|\eta|_{\max}$ from equation (5.6) has no observable meaning, because it lies totally inside the coherence time of the single particle pair creation process, which according to equation (5.1) is now determined by the cosmological expansion only and which is centred around the interval $|\eta|\lesssim(e|E|/mb)(mb)^{-1/2}$ (cf equations (5.6) and (4.3)) with width $(mb)^{-1/2}$ respectively $(ma^2/b)(mb)^{-1/2}$ (cf equation (5.1)). This means, however, that the electromagnetic field is (coherently) immersed in the now cosmologically dominated pair creation mechanism. When $E\to 0$ and $H\to 0$, the influence of the electromagnetic field on the pair creation process vanishes completely and we obtain, as is easily seen from the equations (4.10), (5.1) and (5.6), the pure cosmological pair creation result of Audretsch and Schäfer (1978), which we have already mentioned at the end of § 4. It should be stressed that in our weak field case we were not forced to make a statement about the ratio $|E|/|E_c|$, where $|E_c|/a^2=m^2/e$ is the critical field strength for the Klein 'paradox' at $\eta=0$, if the space–time was flat at $\eta=0$ (remember that $|E|/\Omega^2$ is the absolute value of the observable field strength (cf (2.9)) and that it is largest at $\eta=0$). The reason is that if we have $|E|\gtrsim|E_c|$, then our weak field condition $e|E|\ll mb$ implies $a^2/b\ll ma^2/e|E|\lesssim m^{-1}$, which on the other hand means that a typical radius of curvature of the space–time at $\eta=0$ (according to the preceding discussion the pairs come into existence near $\eta=0$) is much smaller than the corresponding 'radius of curvature' of the electromagnetic field and than the Compton wavelength of the particles, so that for the particles the space–time is extremely non-flat at $\eta=0$ and the critical electromagnetic field strength for the Klein 'paradox' in flat space–time loses its meaning completely. Clearly the case $|E|\lesssim|E_c|$ goes without saying.

6. Cosmological consequences

The application of the foregoing considerations on the cosmological situation is only possible under certain reservations. In particular our calculations are performed using the adiabatic conservation laws (2.4) and (2.9) strongly, which are surely not valid in the earliest stage of the Universe because of the interaction of all the fields present. In order to avoid this lack a new self-consistent calculation taking into account the transition of the different forms of energy would be necessary which will not be performed in this paper. Consequently the following considerations have primary model character only.

Furthermore the electric and magnetic fields $\mathscr{E}$ and $\mathscr{H}$ are uniform, whereas the electromagnetic background in the Universe is an isotropic black-body radiation field. Thus at first the question arises of under which conditions the quasi-stationary fields $\mathscr{E}$ and $\mathscr{H}$ can be identified with the Planckian background radiation or with a part of it.

Certainly we can consider that part of the radiation field as quasi-stationary which does not change during the creation time of a single particle pair. This time is given by

$$\Delta t = \Omega \Delta \eta \tag{6.1}$$

(see (2.1) and (2.6)), where

$$\Delta \eta \simeq \gamma^{-1/2} \tag{6.1a}$$

is the conformal creation time in view of (5.1)†. Assuming that $e|E|/mb \gg 1$, which is to be proved later, we get from (6.1) and (6.1a) with regard to (4.3) and (2.9):

$$\Delta t = (e\mathscr{E})^{-1/2}. \tag{6.2}$$

Then the condition of stationarity of the radiation field for the particle creation process takes the form:

$$\omega < (e\mathscr{E})^{1/2}. \tag{6.3}$$

Only the low-frequency range of the black-body background radiation satisfying (6.3) can be considered as quasi-stationary. This means that the energy content of this part of the background radiation can be identified with the energy density of our quasi-stationary electromagnetic field. Thus we set (k is the Boltzmann constant)

$$\mathscr{E}^2 = \frac{4}{\pi} kT \int_0^{\omega_{\max} = \delta kT} \omega^2 \, d\omega \tag{6.4}$$

where $\omega_{\max}$ means the maximum frequency satisfying (6.3). Furthermore only the Rayleigh–Jeans part of the Planck distribution is used, which will be justified retrospectively by finding $\delta \ll 1$. From (6.4) we get immediately

$$\mathscr{E}^2 = (4/3\pi)(kT)^4 \delta^3 \tag{6.5}$$

and together with $\delta kT = \beta\sqrt{e\mathscr{E}}$, $\beta \lesssim 1$ (see (6.3)) we find:

$$\delta = (4/3\pi) e^2 \beta^4 = 3{\cdot}1 \times 10^{-3} \beta^4. \tag{6.6}$$

Obviously the factor δ representing a measure for the 'stationary' fraction of the background field energy is independent of the cosmological evolution and its value

† As shown later, the second condition of (5.1) is fulfilled automatically in our situation if the first one is valid.

justifies the use of the Rayleigh–Jeans law for all times very well. From (6.5) and (6.6) we obtain finally:

$$\mathscr{E}^2 = (4kT/3\pi)^4 e^6 \beta^{12}. \tag{6.7}$$

In view of (5.9*a*) the temperature of the background radiation must fulfil for a sufficient particle production the condition:

$$T > \frac{3\pi}{4}\frac{m}{e^2 k}\beta^{-3} \triangleq 3{\cdot}5\times 10^{15}\beta^{-3}\ \mathrm{K} \tag{6.7a}$$

if we choose m, as in the following, to be the mass of the proton. This temperature is exceptionally high and may not really be reached in the early Universe as a consequence of the breakdown of the adiabatic conservation laws (2.4). Therefore our calculations have the meaning of a rough model and describe the reality in a satisfactory way only after a better fitting of the temperature behaviour.

Now we are able to calculate the quantity $e|E|/mb$, which is the important one for the particle creation in the case of the presence of uniform electromagnetic fields. With respect to (2.9) we obtain at first:

$$e|E|/mb = e\mathscr{E}\Omega^2/mb. \tag{6.8}$$

From (2.4) and (2.5*a*) it follows that

$$b = \Omega^2 T^2 (8\pi G\sigma/3)^{1/2}. \tag{6.9}$$

Inserting (6.7) and (6.9) into (6.8) we find with the definition of σ:

$$\frac{e|E|}{mb} = \left(\frac{4}{3\pi}\right)^{3/2}\left(\frac{15}{2G}\right)^{1/2}\frac{e^4}{\pi^2 m}\beta^6 = 5{\cdot}3\times 10^{13}\beta^6. \tag{6.10}$$

Evidently the condition $e|E|/mb \gg 1$ made above is fulfilled for a large range of $\beta(\beta \leqslant 1)$.

Thus we have found the most surprising result. In the radiation-dominated Universe a fixed part of the background radiation field characterised by δ (see (6.6)) can be considered as a quasi-stationary electromagnetic field, which influences the particle creation process because of the large value of (6.10) much more than the gravitational field, the influence of which is negligible (cf § 4).

As is well known (Schwinger 1951), in plane electromagnetic waves no particle creation occurs for the case of the flat Minkowski space–time. This result may be transferable to our geometry in view of the large value (6.10) and the expression (5.9). But for the superposition of the different modes of the stationary part of the isotropic background radiation the situation of a plane electromagnetic wave is not given. On the contrary, during the stationary phase determined by (6.2) the angle between the resulting electric and magnetic field strengths can take any value. Therefore our preceding calculation with parallel stationary electric and magnetic fields for the *low* frequency part of the background radiation is justified as a useful approximation†.

Finally we calculate the number density of the created particles. At first the maximum value of time for the particle creation will be determined. It follows from

† For vanishing Poynting vector (strongly isotropic case) the angle between the resulting electric and magnetic field strengths must be zero (or π) without averaging procedure.

(5.6), going over from the conformal time η to the cosmological time t with regard to (2.7b) for the radiation-dominated Universe:

$$t_{\max}=\frac{1}{2}\left(\frac{eE}{mb}\right)^2\frac{b}{\gamma}. \tag{6.11}$$

At this time the pair creation time reaches, according to (5.9a) and (6.2), its maximum value of $\Delta t_{\max}\simeq m^{-1}\triangleq 10^{-24}$ s. The substitution of γ with regard to (4.3) results in view of the large value of (6.10) in

$$t_{\max}=\frac{1}{2}\frac{e|E|}{m^2b}=\left(\frac{4}{3\pi}\right)^{3/2}\left(\frac{15}{2G}\right)^{1/2}\frac{e^4}{2\pi^2m}\beta^6\triangleq 1\cdot 9\times 10^{-11}\beta^6\,\text{s}. \tag{6.12}$$

Accordingly particle creation only happens in the immediate neighbourhood of the 'big bang'. Thereafter the created particles are existent and their number density is according to (4.10), (4.10a) and (2.7b) in view of the magnitude of (6.10) given by:

$$n=\hat{n}/(2bt)^{3/2}$$
$$\hat{n}=\frac{(e|E|)^{5/2}}{(2\pi)^3mb}K\exp\left(-\pi\frac{m^2a^2}{e|E|}\right) \tag{6.13}$$

with $K=0\cdot 27$ for $\mathscr{E}=\mathscr{H}$ in the case of the given fields. Now we have to take into account that at a critical time $t_c(\gg t_{\max})$ the radiation-dominated Universe goes over into the matter-dominated era. In this region the expansion law has, for flat three-dimensional space (see (2.1)), the form

$$\Omega\sim t^{2/3}. \tag{6.14}$$

From the conservation laws for the energy of matter ($\rho_m\Omega^3=$ constant) and radiation ($\rho_r\Omega^4=$ constant) it follows immediately by the use of (6.14) that

$$\rho_{0_m}/\rho_{0_r}=\Omega_0/\Omega_c=(t_0/t_c)^{2/3}. \tag{6.15}$$

Here ρ_0, Ω_0 and t_0 represent the present values of energy density, distance parameter and time, whereby

$$t_0=\tfrac{2}{3}H_0^{-1}=3\cdot 7\times 10^{17}\,\text{s}=12\times 10^9\,\text{yr} \tag{6.15a}$$

($H_0=55$ km s^{-1} Mpc^{-1}, the present Hubble parameter) is the age of the Universe. Thus we find from (6.15):

$$t_c=t_0\left(\frac{\rho_{0_r}}{\rho_{0_m}}\right)^{3/2}=\tfrac{2}{3}H_0^{-1}\left(\frac{8\pi G\sigma T_0^4}{3H_0^2}\right)^{3/2}=7\times 10^{-17}t_0=2\times 10^{11}\,\text{s}=8\times 10^3\,\text{yr} \tag{6.16}$$

wherein the relation

$$\rho_{0_m}=(3/8\pi)H_0^2/G\triangleq 6\times 10^{-30}\,\text{g cm}^{-3} \tag{6.16a}$$

valid for the flat expanding Universe has been inserted ($T_0=2\cdot 7$ K, present temperature of the background radiation).

Because of the conservation law of the particle number in the matter-dominated era we have, in view of (6.14), the following relation for the present number density of the created particles:

$$n_0=n_c(t_c/t_0)^2 \tag{6.17}$$

with

$$n_c = \hat{n}/(2bt_c)^{3/2} \tag{6.17a}$$

as the number density at the point of the critical time t_c (see (6.13)). Insertion of (6.16) and (6.17a) into (6.17) yields with respect to (6.13):

$$n_0 = \frac{0\cdot27}{(2\pi)^3}\left(\frac{e|E|}{mb}\right)^{5/2}(3\pi m^2 G\sigma/2)^{3/4} T_0^3 \exp\left(-\pi\frac{m^2a^2}{e|E|}\right). \tag{6.18}$$

With $e|E|/mb$ according to (6.10) we obtain finally:

$$n_0 = \frac{0\cdot27\times2^{5/2}\sqrt{15}}{3^3\pi^{19/2}}\frac{e^{10}}{mG^{1/2}}(kT_0)^3\beta^{15}\exp\left(-\pi\frac{m^2a^2}{e|E|}\right)$$

$$\hat{=} 1\cdot8\times10^6\beta^{15}\exp\left(-\pi\frac{m^2a^2}{e|E|}\right)\ \mathrm{cm}^{-3}. \tag{6.19}$$

As one can simply prove with the use of (2.4), (2.5a) and (6.10), the quantity $m^2a^2/e|E|$ is small compared to one. According to this it is also justified retrospectively to calculate with the first expression (5.1) as before. Thus the result (6.19) is simplified to

$$n_0 = 1\cdot8\times10^6\beta^{15}\ \mathrm{cm}^{-3}. \tag{6.20}$$

This result is also valid in the case of vanishing initial matter ($a = 0$, cf (2.4) and (2.5a)). Then all particles are produced in consequence of the electromagnetic field in the expanding Universe.

7. Conclusions

Evidently the value of the created particle density given by (6.20) is of such an order that the particle creation mechanism discussed above cannot be neglected in cosmology. Unfortunately our result depends very sensitively on the value of the parameter β, so that a precise comparison with the observation is impossible. However the present mean particle density in the Universe lies inside the range of the result (6.20). For $\beta = 0\cdot15$ we find the present mean particle number density in the Universe of about 10^{-6} cm^{-3}. On the other hand, neglecting the influence of the electromagnetic fields one gets a ratio between created and present particles of only about 10^{-20} (cf Mamaev *et al* 1976).

Consequently, in cosmology the particle creation out of the vacuum by non-gravitational fields is important and the back-reaction of the created particles on the metric should be taken into account, for instance by the use of the particle creation rate (5.9). In this connection we note that, besides the creation mechanism discussed in this paper, the strong interaction fields, introduced at first in (2.3) but neglected in the particle field equation (3.1), should also give an additional amount for created particles as well as the high frequency part of the background radiation.

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